On the Possibility of Observing the Shapiro Effect for Pulsars in Globular Clusters

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For pulsars in globular clusters, we suggest using observations of the relativistic time delay of their radiation in the gravitational eld of a massive body (the Shapiro effect) located close to the line of sight to detect and identify invisible compact objects and to study the distribution of both visible and dark matter in globular clusters and various components of the Galaxy. We have derived the dependences of the event probability on the Galactic latitude and longitude of sources for two models of the mass distribution in the Galaxy: the classical Bahcall-Soneira model and the more recent Dehnen-Binney model. Using three globular clusters (M15, 47 Tuc, Terzan 5) as an example, we show that the ratios of the probability of the events due to the passages of massive Galactic objects close to the line of sight to the parameter f2 for pulsars in the globular clusters 47 Tuc and M15 are comparable to those for close passages of massive objects in the clusters themselves and are considerably higher than those for the cluster Terzan 5. We have estimated the rates of such events. We have determined the number of objects near the line of sight toward the pulsar that can produce a modulation of its pulse arrival times characteristic of the effect under consideration; the population of brown dwarfs in the Galactic disk, whose concentration is comparable to that of the disk stars, has been taken into account for the first time.

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INTRODUCTION

Detection of the cosmic microwave background anisotropy (WMAP)and investigation of the spatial distribution of galaxies and their clusters (SDSS) (Tegmark et al. 2004) have allowed the most probable values of the most important cosmological parameters, such as the baryon density, the total matter density, and the cosmological constant, to be obtained. Analysis of the observational data has revealed that the visible matter accounts for only a few percent of the total mass of the Universe and the observation of dark matter and the determination of its nature are among the key problems in cosmology. Analysis of the rotation curves for spiral galaxies from Rubin et al. (1980) indicated that the mass of the galaxies required to explain these curves is a factor of 10 larger than the total mass of all the known spiral galaxy components (stars, gas, dust). In addition, Ostriker et al. (1974) showed that the galactic disks are unstable without a massive halo whose composition is unknown. These conclusions are also valid for our Galaxy; it is believed that its disk does not contain any signi cant amount of dark matter, while more than 90% of the halo matter is unobservable (Kuijken and Gilmore 1991; Roulet and Mollerach 1997). This dark halo may consist of both baryonic and nonbaryonic matter. In the former case, massive compact halo objects (MACHOs), such as brown dwarfs, primordial black holes, white dwarfs, Jupiter-like planets, etc., may constitute the dark halo mass (Carr 1994). In the latter case, the nonbaryonic matter may be represented by weakly interacting massive particles (WIMPs) that are also capable of clumping into compact dark matter objects (Berezinskiy et al. 2003).

The problem of dark matter detection also exists on smaller scales than the scale of the Universe. As was shown by Heggie and Hut (1996), low-mass stars and white dwarfs, which are unobservable because of their low luminosity, constitute half of the mass of globular star clusters. In contrast to the population of light objects, heavy stars tend to sink to the cluster core as a result of mass stratification. Consequently, one might expect the dark component of a cluster to be detected on its periphery. The presence of a significant amount of dark matter forces the researchers to search for methods of its detection.

One of these methods of searching for dark mass in the Galaxy is to use the characteristic variability of the light curves for certain stars in the Large and Small Magellanic Clouds (Paczynski 1986), i.e., to search for microlensing events. The necessity of monitoring millions of stars and the fact that the detection probability of microlensing events is low are the main problems of this method. The possibility of using extragalactic optical pulsars in searching for dark galactic objects was also discussed (Schneider 1990). When the signal from a pulsar passes near a massive compact object, its ux is magnified by gravitational lensing. However, apart from flux magnification, a time delay of the signal will also be observed in this case (Krauss and Small 1991). Thus, in addition to gravitational lensing, another general relativity effect, namely, the relativistic time delay of the signal from a pulsar in the gravitational field of a massive body can be used to detect both baryonic and nonbaryonic dark matter objects, which was suggested by Larchenkova and Doroshenko (1995). The relativistic travel time delay of an electromagnetic signal in a static, spherically symmetric gravitational field of a point mass is called the Shapiro effect. This, in turn, allowed the parameters of the gravitating body to be determined. Subsequently, Wex et al. (1996)

considered the possibility of using pulsars behind the Galactic center as a test for studying the mass distribution in the Galactic bulge; Fargion and Conversano (1997) described the gravitational Shapiro phase shift on pulsar's period for the detection of the dark matter. The Shapiro effect that results from the passage of a star close to the line of sight to the pulsar was suggested as a possible cause of the pulsar glitches (Sazhin 1986).

The detection of a significant number of pulsars in the globular clusters 47 Tuc, Terzan 5, and M15 (Freire 2006) makes it interesting to discuss the question of using them to study the dark matter both in the globular clusters themselves and in the Galaxy along the line of sight toward these clusters. The lowfrequency pulsar timing noise in globular star clusters produced by the Shapiro effect and attributable to random passages of cluster stars near the line of sight to the pulsar was considered by Larchenkova and Kopeikin (2006). These authors also obtained spectral characteristics of this noise process, in particular, the slope of the power spectrum. The latter will allow the noise due to the stochastic Shapiro effect to be distinguished from the low-frequency timing noise of a different nature in the observational data. However, the detection probability of individual relativistic pulsar signal time delay events in globular clusters caused by the passages of both visible and dark objects of the globular cluster itself and Galactic compact objects close to the line of sight remains unclear.

In this paper, we investigate the possibility of observing and detecting single relativistic signal time delay events for pulsars in globular star clusters. In the next section, we give a brief overview of the signal magnification and angular splitting results for the lensing of a point mass and obtain, in the gravitational lens approximation, numerical estimates of the pulsar pulse time delay in the gravitational field of this mass. Next, we provide brief information about the pulsars in globular star clusters and summarize the calculations of the probability of observing relativistic signal time delay events for pulsars in the clusters 47 Tuc, Terzan 5, and M15 for large impact parameters compared to the Einstein-Chwolson radius both on objects of the clusters themselves and on Galactic objects. In the Sections "Number of Stars" and "The Event Rate", we estimate the number of objects located close to the line of sight to the source and producing the characteristic PAT modulation and the rates of these events, respectively. The results obtained are discussed in Conclusion.

ANGULAR SPLITTING, MAGNIFICATION, AND TIME DELAY OF THE PULSAR SIGNALS

Let us consider the classical model of gravitational lensing for a point lens (a deflecting body) with mass M. The geometry under consideration is presented in Fig. 1. The geometrical position of the pulsar (PSR) in the sky is specified by the angle θ_s, while the positions of its images are specified by the angles θ_+ and θ_-, where the '+' and '−' signs correspond to the first (+) and second (−) images, respectively, d is the impact parameter of the undeflected light ray. The Einstein-Chwolson radius is defined by the formula (Einstein 1965)

$$R_E = (4GMD_{ds}D_d/c^2D_s)^{1/2}, \qquad (1)$$

where c is the speed of light in a vacuum, G is the gravitational constant, D_s is the distance from the pulsar (PSR) to the observer (O), D_{ds} and D_d are the distances from the pulsar to the massive body (M) and from this body to the observer, respectively. Three effects are well known for the classical model under consideration: (1) pulsar image splitting in the plane of the gravitating body, (2) flux magnification, and (3) sign al time delay. For microlensing (lensing by stars), the angular image splitting is much smaller than 10^{-3} arcsec and it cannot yet be resolved with currently available instruments. The flux magnification for the lensed images is given by the formula (Refsdal 1964):

$$\mu_{+,-} = \frac{1}{4}\left[\frac{f}{(f^2+4)^{1/2}} + \frac{(f^2+4)^{1/2}}{f} \pm 2\right], \tag{2}$$

where f is the dimensionless impact parameter, $f = d/R_E$. It is easy to see from Eq. (2) that significant pulsar flux magnification takes place only if the impact parameter d falls into the EinsteinChwolson ring, i.e., $d < R_E$. When $f \gg 1$, the contribution from the second $(-)$ image to the total brightness is small and this image is too faint to be observable.

Strict pulsar pulse periodicity allows the PAT variations to be used to detect the massive bodies located close to the signal propagation path from the pulsar to the observer. The coordinate travel time delay of the light ray along the trajectory deflected by a gravitating body relative to the undeflected trajectory is given by the formula (Schneider et al. 1992)

$$c\Delta t = \frac{D_d D_{ds}}{2D_s}[\hat{\alpha}(\xi)]^2 - \hat{\psi}(\xi) + const, \tag{3}$$

where $\hat{\psi}$ is the deflecting potential, $\hat{\alpha}$ is the deflection angle, and ξ specifies the ray location in the plane of the gravitating body. As we see from this formula, the time delay consists of three components: the geometrical time delay (the first component), the relativistic time delay (the second component), and the constant calculated by Kopeikin and Schafer (1999). For a spherically symmetric Schwarzschild lens, Eq. (3) can be rewritten as

$$\Delta t = \frac{2GM}{c^3}\left[\frac{4}{(\sqrt{f^2+4} \pm f)^2} - \ln(\sqrt{f^2+4} \pm f)^2\right] + const. \tag{4}$$

Since we are interested in the case of $f \gg 1$, for which, as was mentioned above, only one pulsar image (signal) is observed, it follows from Eq. (4) that the geometrical time delay of the first ray $(+)$ may be neglected compared to the relativistic time delay. Since the pulsar and the massive body are moving, it is convenient to estimate the Shapiro effects in the gravitational lens approximation in terms of the pulsar velocity V_P projected onto the plane of the sky relative to the massive body (M) and the time of the closest approach of this body to the line of sight (T_0) . The relativistic signal time delay can then be written as (Larchenkova and Doroshenko 1995)

$$\Delta t = -\frac{2GM}{c^3} \cdot \ln(1 + {\beta_0}^2 \cdot (t - T_0)^2), \tag{5}$$

where we use the designation $\beta_0 \equiv V_P/d$. Let the pulsar PAT observation begin at time $t = t_0$. The maximum relativistic time delay occurs at the $t = T_0$ that corresponds to

Table 1: Observing time $(t_0 - T_0)$ it takes for the Shapiro effect delay $\Delta t_{min} = 0.5 \mu s$ to be recorded for various masses of the gravitating body M and impact parameters d.

M	V_P=30 km s^{-1}		V_P=200 km s^{-1}	
	d=10 AU	d=100 AU	d=10 AU	d=100 AU
$1 M_\odot$	128 days	3.5 years	20 days	197 days
$0.5 M_\odot$	179 days	4.9 years	28 days	281 days
$0.1 M_\odot$	1 year	10 years	69 days	1.9 days

the minimum impact parameter, which is a function of time. Suppose that the minimum measurable Shapiro delay is 0.5 μs, which corresponds to the current status of millisecond pulsar timing. For our case of large impact parameters, the observing time $(t_0 - T_0)$ it takes for the specified relativistic time delay to be recorded is given in Table 1 for various masses of the deflecting body, impact parameters, and pulsar velocities relative to the gravitating body. Note that the velocities used in the table are close to the typical velocities of the objects in globular clusters and the Galaxy, respectively. The corresponding maximum Shapiro delays that can be observed in the characteristic time $t_0 - T_0 = 5$ years are given in Table 2 for various gravitating masses, impact parameters, and relative pulsar velocities. Let us

Table 2: Maximum Shapiro delay observed in 5 years for variousmasses of the gravitating body M and impact parameters d.

M	V_P=30 km s^{-1}		V_P=200 km s^{-1}	
	d=10 AU	d=100 AU	d=10 AU	d=100 AU
$10 M_\odot$	237 μs	10 μs	541 μs	129 μs
$1 M_\odot$	23.7 μs	1 μs	54.1 μs	12.9 μs
$0.5 M_\odot$	11.9 μs	0.5 μs	27 μs	6.5 μs
$0.1 M_\odot$	2.4 μs	0.1 μs	5.4 μs	1.3 μs

introduce a new quantity, d_{max}, the maximum value of the impact parameter d at which the relativistic time delay is still observable in the pulsar PAT residuals. As follows from Eq. (5), d_{max} is a function of the mass of the deflecting body M, the relative pulsar velocity V_P, and the minimum observable Shapiro delay δt_{min}. Consider two possibilities: the massive body is located in a globular star cluster (1) and in the Galaxy along the line of sight it (2). Let the observing time $(t_0 - T_0) = 5$ years, $\delta t_{min} = 0.5 mus$, $V_P = 30$ km s^{-1} in the first case and $V_P = 200$ km s^{-1} in the second case. We use the designations d_{max1} and d_{max2} corresponding to these two cases. The derived expressions for d_{max1} and d_{max2} depend only on the mass of the deflecting body and can be well approximated in a wide range of masses

by a function of the form $d \simeq Am^{\alpha}$ with the following parameters:

$$d_{max1}[a.e.] \simeq \begin{cases} 138 \cdot m^{0.45}, 0.08 \leq m < 0.5 \\ 142 \cdot m^{0.5}, \quad 0.5 \leq m < 10 \end{cases}$$

$$d_{max2}[a.e.] \simeq \begin{cases} 922 \cdot m^{0.45}, 0.08 \leq m < 0.5 \\ 948 \cdot m^{0.5}, \quad 0.5 \leq m < 10 \end{cases} \tag{6}$$

where m is the mass of the deflecting body in solar masses. Since the number of stars decreases rapidly with increasing stellar mass (see the mass function in the Section "The Probability of a Single Relativistic Delay Event"), we limited the range of masses under consideration from above by $10 M_{\odot}$.

PULSARS IN GLOBULAR CLUSTERS

Twenty four globular star clusters containing 129 pulsars are known to date in our Galaxy (Freire 2006). The overwhelming majority of these pulsars have spin periods shorter than 25 ms and more than two thirds of them are members of binary systems. Respectively, 8 (Anderson 1992), 22 (Lorimer et al. 2003), and 32 (Ransom et al. 2005) pulsars have been discovered in the globular clusters 15 (NGC 7078), 47 Tuc (NGC 104), and Terzan 5. All of the pulsars in 15, except for B2127+11C that is a member of a binary system, are concentrated in the cluster core (Anderson 1992). In addition, all of the pulsars in 47 Tuc are within 1.2 arcmin of the cluster center (i.e., within three cluster core radii)and have spin periods up to 8 ms (Lorimer et al. 2003). The globular star clusters in which pulsars have been discovered have a dense core and, in most cases, a large total mass, which manifests itself in high star escape velocities from the cluster. For example, the escape velocity v_{esc} is 58 km s^{-1} for 47 Tuc and 55 km s^{-1} for M15 (Webbink 1985; Gebhardt al.1997). The mean proper motion of the millisecond pulsars is 87 ± 13 km s^{-1}, which is a factor of 3 smaller than that of the normal pulsars (Hobbs et al. 2005). The proper motions were measured for some of the pulsars in globular clusters. For example, the mean 2D pulsar speed for 11 pulsars in 47 Tuc corrected for the proper motion of the cluster itself is 25 ± 5 km s^{-1} (Hobbs et al. 2005), in good agreement with the escape velocity from the cluster (58 km s^{-1}). Based on these measurements, we used the transverse pulsar velocity of 30 km s^{-1} in our estimates.

THE PROBABILITY OF A SINGLE RELATIVISTIC DELAY EVENT

If the optical depth of the effect is so small that the event occurs only once, then the differential probability of the event can be written as $dP = n(x)\pi(fR_E)^2 dx$, where $n(x)$ is the number density of deflecting bodies with mass M (Krauss and Small 1991). The probability of a relativistic delay on all of the massive objects between the pulsar and the observer can then be written as:

$$P = \int_0^{D_s} \frac{4\pi G}{c^2} f^2 \rho(x) \frac{(D_s - x)x}{D_s} dx \tag{7}$$

Table 3: Basic parameters of the globular clusters M15, 47 Tuc, and Terzan 5.

	M15	47 Tuc	Terzan5
$l;\ b\ (°)$	65.01; -27.31	305.9; -44.89	3.84; 1.67
D_c (kpc)	10.2	4.1	10.3
r_c (pc)	0.07	0.52	0.40
σ (/)	11.6	11	10.6
$\rho_{0,gc}$ $(M_\odot \mathrm{pc}^{-3})$	2×10^6	6×10^4	5×10^5
r_t (pc)	60.8	60.3	21.9
D_{GC} (kpc)	11.2	9.1	1.0

where $\rho(x) = n(x)M$. Note that this probability depends on the total mass of all deflecting bodies and does not depend on the mass of an individual object. Let us now calculate the probabilities of a signal delay event for two cases: (1) the pulsar and the massive body are located in a globular star cluster; (2) the pulsar is still located in the cluster, while the massive body is located in the Galaxy along the line of sight to the cluster. The latter case is also applicable to the Galactic pulsars that do not belong to globular star clusters.

The Pulsar and the Massive Body in the Globular Cluster

Let us consider the globular cluster as a selfgravitating isothermal sphere of identical stars (see, e.g., the calculations by Jetzer et al. (1998)f or 47 Tuc). Basic parameters of the clusters M15, 47 Tuc, and Terzan 5 considered here are given in Table 3, where l and b are the Galactic coordinates, D_c is the distance to the cluster center, r_c is the cluster core radius, σ is the stellar velocity dispersion of the cluster, $\rho_{0,\mathrm{gc}}$ is the central density of the cluster core, r_t is the tidal radius, and D_{GC} is the distance from the Galactic center (GC) to the cluster (Gebhardt et al. 1997;Webbink 1985; Jetzer et al. 1998). To estimate the probability of signal time delay events, we will use a simple cluster model in which the mass density as a function of the cluster radius R is given by

$$\rho_{\mathrm{gc}}(R) = \frac{\rho_{0,\mathrm{gc}}}{1 + \left(\frac{R}{r_c}\right)^2}. \tag{8}$$

The event probability on cluster objects will then be written as the following integral (see Eq. 7)

$$P = \int_{x_t}^{D_s} \frac{4\pi G}{c^2} f^2 \rho_{\mathrm{gc}}(x) \frac{(D_s - x)x}{D_s} \, dx \tag{9}$$

where the integration terminates at the cluster boundary $x_t = D_c - \sqrt{r_t^2 - D_c^2 \sin\beta^2}$. Here, β is the angle between the directions of the cluster center and the source (for small angles considered in our problem, $\sin\beta \simeq \beta$).

The ratio of the probability of a relativistic delay event to the square of the dimensionless impact parameter f, P/f^2, as a function of the source position in the globular cluster along the observer's line of sight to the cluster center (i.e., at $\beta = 0$) depending on the radial distance to the cluster center is indicated by different curves in Fig. 2 for each of the clusters under consideration. The characteristic dependence of P/f^2 on the angular distance between the directions of the source and the cluster center, i.e., in a direction perpendicular to the line of sight, is shown in Fig. 3 for a pulsar at the distance of the center of the globular cluster Terzan 5. For clarity, Fig. 4 shows the 3D distribution of the ratios of the probability of signal time delay events to the square of the dimensionless impact parameter f as a function of the angular and radial distances to the cluster center for Terzan 5.

If the pulsar lies at the globular cluster center, i.e., $\beta = 0$ and $D_s = D_c$, then P/f^2 is 4.65×10^{-8} for 47 Tuc, 4.01×10^{-8} for M15 and 1.92×10^{-7} for Terzan 5. To calculate the event probability itself, we must know the impact parameter f, which, in turn, is a function of the mass of the deflecting body and d_{max1}. For the subsequent estimates, we will make the following significant simplifying assumptions: the mass function for globular cluster stars does not depend on the positions of the stars in the cluster and all of the cluster objects deflecting the electromagnetic signal from the pulsar have the same mass $0.3 M_\odot$ or $0.6 M_\odot$. These values are variously estimated (see, e.g., Chabrier 2003 and references therein) to be close to the mean masses of the globular cluster stars and, hence, using them in our calculations seems quite justifiable and close to reality. The latter assumption is also confirmed, for example, by the fact that the population of low-mass white dwarfs in 47 Tuc accounts $\sim 50\%$ of the total cluster mass (Heggie and Hut 1996), while the white dwarfs in M15 constitute the vast bulk (in mass)of the entire cluster population (about 85%; Gebhardt et al. 1997).

Let us consider the dimensionless impact parameter f ($f \gg 1$)as a function of the distance between the deflecting body with a mass of $0.3 M_\odot$ or $0.6 M_\odot$ and pulsar that are both located in the globular cluster for two impact parameters, $d = 10$ and $d = 80$ AU. In Fig. 5, these dependences are shown in a wide range of distances covering the linear sizes of all three clusters under consideration ($0 - 120$ pc). Assuming the distance between the massive body and the pulsar to be half the tidal cluster radius, we will obtain, as an example, the following estimates for the probability of a pulsar signal time delay event for the deflecting body with a mass of $0.6 M_\odot$ and the impact parameter $d = 80$ AU: $P \sim 2 \times 10^{-3}$ for 47 Tuc, $P \sim 1.7 \times 10^{-3}$ for M15, and $P \sim 2.5 \times 10^{-2}$ for Terzan 5 (the pulsar is located at the cluster center). For the deflecting body with a mass of $0.3 M_\odot$, our estimates increase by a factor of ~ 1.4. Note also that since the matter in the globular clusters concentrate strongly to the cluster center, the characteristic distance between the deflecting body and the pulsar is much less than half the tidal radius, which also causes the probability of such events to increase significantly (see Fig. 5).

The Pulsar in the Cluster and the Massive Body outside the Cluster

Let us now consider the case where the pulsar is in the globular cluster, while the deflecting body is outside the cluster. As was noted above, the calculations and estimates given below also remain valid for the pulsars that do not belong to globular clusters. Inaccurate knowledge of the parameters of the Galactic components (disk, bulge, spheroid, halo, etc.)poses the main difficulty in calculating the probability of relativistic delay events on Galactic objects. A four-component model of the Galaxy consisting of a disk, a central bulge, a spheroid, and a halo was first suggested by Bahcall and Soneira (1980) and Bahcall (1986). In some way, this model is "classical" and is still commonly used in many calculations, including the calculations of weak lensing events (see, e.g.,Wex et al. 1996). A large amount of observational data in various wavelength ranges has appeared in recent years. These data have made it possible to improve significantly our knowledge of the Galactic structure and the parameters of the models that describe it (see, e.g., Dwek et al. 1995; Dehnen and Binney 1998; Robin et al. 2003). Our subsequent calculations of the probability of relativistic pulsar signal delay events, the number of massive bodies near the line of sight, and the event rate for Galactic objects are based on one of the most commonly used models suggested by Dehnen and Binney (1998)(below referred to as DB). This model includes a three-component disk that consists of thin and thick disks and an interstellar medium, a flatted bulge, and a halo in the following form:

$$\rho_{\text{disk}}(r, z) = \rho_{0,\text{disk}} \exp\left[-\frac{R_m}{r} - \frac{r}{R_d} - \frac{|z|}{z_d}\right]$$

$$\rho_{\text{bulge}}(r, z) = \rho_{0,\text{bulge}} \left(\frac{\sqrt{r^2 + \frac{z^2}{q_b^2}}}{r_{0,b}}\right)^{-1.8} \exp\left[-\frac{r^2 + \frac{z^2}{q_b^2}}{r_{t,b}}\right]$$

$$\rho_{\text{halo}}(r, z) = \rho_{0,\text{halo}} \frac{r^2 + \frac{z^2}{q_h^2}}{r_{0,h}^2} \left(1 + \frac{r^2 + \frac{z^2}{q_h^2}}{r_{0,h}}\right)^{-4.207}, \tag{10}$$

where $\rho_{\text{disk}}(r, z)$ describes each of the three disk components with its own parameters $\rho_{0,\text{disk}}$, R_m, R_d, z_d and the total mass $M_{\text{disk}} \simeq 4.8 \times 10^{10} M_\odot$; $\rho_{0,\text{bulge}} = 0.7561 M_\odot/\text{pc}^3$, $r_{0,b} = 1$ kpc, $r_{t,b} = 1.9$ kpc, $q_b = 0.6$, $\rho_{0,\text{halo}} = 1.263 M_\odot/\text{kpc}^3$, $r_{0,h} = 1.09$ kpc, $q_h = 0.8$.

For comparison, we also calculated the probabilities of relativistic signal delay events for the "classical" model by Bahcall and Soneira (below referred to as BS):

$$\rho_{\text{disk}}(r, z) = \rho_{0,\text{disk}} \exp\left[\frac{R_{GC} - r}{3.5 \text{ kpc}} - \frac{|z|}{125\text{pc}}\right]$$

$$\rho_{\text{bulge}}(r) = \rho_{0,\text{bulge}} \left(\frac{r}{1 \text{ kpc}}\right)^{-1.8} \exp\left[-\left(\frac{r}{1 \text{ kpc}}\right)^3\right]$$

$$\rho_{\text{spher}}(r) = \rho_{0,\text{spher}} \frac{\exp\left[-b\left(\frac{r}{2.8 \text{ kpc}}\right)^{1/4}\right]}{\left(\frac{r}{2.8}\right)^{7/8}}$$

$$\rho_{\text{halo}}(r) = \rho_{0,\text{halo}} \frac{a^2 + R_{GC}^2}{a^2 + r^2}, \tag{11}$$

where R_{GC} is the solar Galactocentric distance (assumed to be 8 kpc), $\rho_{0,\text{disk}} = 0.04 M_\odot/^3$ is the disk density in the solar neighborhood, the bulge mass is $M_{\text{bulge}} \simeq 10^{10} M_\odot$, $\rho_{0,\text{spher}} \simeq 1/500 \rho_{0,\text{disk}}$, $b = 7.669$, $\rho_{0,\text{halo}} = 0.01 M_\odot/\text{pc}^3$, a is the core radius of the spherical dark matter halo. Its value is believed to lie in the range from ~ 2 to ~ 8 kpc (Caldwell and Ostriker 1981; Bahcall et al. 1983). Since this uncertainty has a negligible effect on the estimate of the effect, we assume in our estimates that $a \simeq 2$ kpc.

It is easy to see from Eqs. (10) and (11) that the Galactic bulge contributes significantly to the probability only for the pulsars whose electromagnetic signal propagates in the immediate vicinity ($\lesssim 1 - 2$ kpc) of the Galactic center. It follows from Table. 3, which presents basic parameters of the globular clusters under study, including the GC distance D_{GC}, that it is important to take into account the influence of the bulge only for the pulsars in Terzan 5, while the signal from the pulsars in the high-latitude clusters M15 and 47 Tuc propagates toward the observer outside the bulge. Note also that the contribution from the disk naturally decreases with increasing Galactic latitude.

Figure 6(a,b) shows the ratios of the detection probability of relativistic PAT time delay events to f^2 as a function of the Galactic coordinates of the source for two distances between the observer and the pulsar, 10.2 and 4.1 kpc (the DB model of the Galaxy). These distances were chosen, because the latter matches the distance to 47 Tuc, while the former roughly corresponds to the distances to M15 and Terzan 5 (the positions of all three clusters are indicated by the crosses and the asterisk). We see that the detection probability of events for large distances increases significantly as the Galactic center is approached due to an enhanced concentration of objects in this region. Note, however, that we excluded the innermost part of the bulge called the "Nuclear Bulge" within ~ 30 pc.

Since a supermassive black hole is present at the Galactic center, the structure of this region is rather complex and cannot be described by a single component (for more detail, see Launhardt et al. 2002). Figure 6(c) shows the contributions from various Galactic components to the total detection probability of events for a source at a distance of 10.2 kpc and a Galactic latitude of $1.67°$ (the latitude of the globular cluster Terzan 5). We see from the figure that the contribution from the bulge is significant only at $|l| \lesssim 20°$.

For comparison, Fig. 7 shows the same dependences as those in Fig. 6 for the BS model of the Galaxy. On the whole, the picture is similar to what has been obtained previously for the DB model. We only note that the contribution from the spheroid is negligible compared to the other components and the influence of the bulge is significant in a narrower l range. The slightly higher probabilities for the BS model (by $sim 10 - 40\%$) are attributable to a more compact bulge at the same total mass $\sim 10^{10} M_\odot$ as that in the DB model and a more massive (by a factor of $sim 1.5$) Galactic disk assumed in the BS model than that in the DB model.

To conclude this section, note that the structure of the Galaxy and its components is assumed to be axisymmetric in both models under consideration. In contrast, observations (see, e.g., Dwek et al. 1995; Revnivtsev et al. 2006) suggest that there is a small asymmetry ($sim 10 - 15\%$) in the distribution of stars relative to the Galactic center related to a rotated,

elongated bulge in the central regions of the Galaxy.

THE NUMBER OF STARS

We can now calculate the expected number (N) of visible stars on the pulsar signal propagation path to the observer that affect the PATs. The mass functions for the stars of various Galactic components and the globular cluster were taken from Chabrier and Mera (1997)and Chabrier (2003) in the form the disk, bulge

$$
\xi(\log m) = \frac{1}{\log M_\odot \mathrm{pc}^3}
\begin{cases}
0.158 \exp\left[-\frac{\left(\log\left(\frac{m}{0.079}\right)\right)^2}{0.9577}\right], 0.08 \le m < 1 \\
4.4 \times 10^{-2} m^{-4.37}, 1.0 \le m < 3.47 \\
1.5 \times 10^{-2} m^{-3.53}, 3.47 \le m < 10.0.
\end{cases}
\tag{12}
$$

halo

$$
\xi(m) = 4 \times 10^{-3} \left(\frac{m}{0.1 M_\odot}\right)^{-1.7} \frac{1}{M_\odot \mathrm{pc}^3}, 0.01 \le m < 0.8
\tag{13}
$$

globular cluster

$$
\xi(\log m) = \frac{1}{\log M_\odot \mathrm{pc}^3}
\begin{cases}
3.6 \times 10^{-4} \exp\left[-\frac{\left(\log\left(\frac{m}{0.33}\right)\right)^2}{0.2312}\right], m \le 0.9 \\
7.1 \times 10^{-5} m^{-1.3}, m > 0.9.
\end{cases}
\tag{14}
$$

where the mass m is in solar masses. It is easy to see that the above formulas describe the stellar component of the Galaxy, i.e., the objects with masses $> 0.08 M_\odot$. However, a large population of objects with masses $0.01 < m \lesssim 0.08 M_\odot$, the so-called brown dwarfs, is currently believed to be present in the Galaxy (particularly in its disk). Whereas the contribution from such objects to the total mass of the Galaxy is small, they can contribute noticeably to the estimate of the number of stars affecting the PATs, since their concentration is comparable to that of the normal stars. The mass function for brown dwarfs is (Chabrier 2003)

$$
\xi(\log m) = 0.158 m^0 \frac{1}{\log M_\odot{}^3}, 0.01 \le m < 0.08.
\tag{15}
$$

The number of Galactic stars affecting significantly the PATs can be written as

$$
N = \frac{\int_0^{D_s} \int_{m_1}^{m_2} \pi \rho(x) \xi(m) [d_{max2}(m)]^2 \, dx \, dm}{M_\odot \int_{m_1}^{m_2} m \xi(m) \, dm},
\tag{16}
$$

where $\rho(x)$ the density of the sources in the Galaxy (the DB model) along the line of sight to the pulsar, $(m1, m2)$ is the mass range of objects under consideration, in our case, from 0.01 to $10 M_\odot$ if the brown dwarfs are taken into account. Equation (16) is valid for calculating the number of stars affecting the pulsar PATs and belonging to the globular cluster itself to within the substitution of d_{max1} for d_{max2}. Integrating Eq. (16) in the entire range of masses and distances and taking into account Eqs. (10) and (12)(15), we obtain estimates for the

Table 4:

	disk	bulge	halo	Galaxy	cluster
M15	0.003	-	0.009	0.012	0.336
47 Tuc	0.002	-	0.004	0.006	0.075
Terzan 5	0.273	0.166	0.039	0.478	0.472

expected number of objects located close to the line of sight both in the Galaxy (given the contribution from its various components) and in the globular cluster (see Table 4).

We see that the expected number of Galactic objects near the line of sight for the globular clusters M15 and 47 Tuc located far from the Galactic center is considerably smaller than the number of objects in the globular cluster itself. The two numbers are equal only for the cluster Terzan 5 located immediately behind the Galactic center. Note also the larger relative contribution from halo objects than that from disk ones for the high-latitude clusters M15 and 47 Tuc.

It should be noted that one of the microlensing events detected by the MACHO group is most likely caused by a lens located in the disk (Alcock et al. 2001) and observed with the Hubble Space Telescope (HST). The observations yielded estimates of the lens mass: this mass either is $\sim 0.04 M_{\odot}$ or lies in the range $0.095 - 0.13 M_{\odot}$, i.e., the lens is either a brown dwarf or a low-mass star.

THE EVENT RATE

Undoubtedly, the detection probability of relativistic PAT time delay events (or, in other words, the optical depth)is a very important quantity. However, knowing the rate of such events is no less important. By analogy with the rate of lensing events (Griest 1991), we will introduce the differential number of pulsar PAT time delay events

$$dN_{ev} = N_s t_{obs} d\Gamma, \tag{17}$$

where N_s is the total number of sources (pulsars) for an observing time t_{obs}, $d\Gamma$ is the differential event rate,

$$d\Gamma = \frac{n(\mathbf{x}) f(\mathbf{v}_l) d^3 x d^3 v}{dt}, \tag{18}$$

where the numerator is the number of massive bodies in the element of volume $d^3 x = dx dy dz$ and velocity space $d^3 v = dv_x dv_y dv_z$ relative to the position $\mathbf{x}$ in the event "tube", $n(\mathbf{x})$ is the number density of massive objects, and $f(\mathbf{v}_l)$ is their velocity profile. The differential velocity with which the gravitating masses with velocities $\mathbf{v}_l$ contribute to the event tube at the position $\mathbf{x}$ and the cylindrical segment $ld\phi = d_{max} d\phi$ can be written as

$$d\Gamma = \frac{\rho(x)}{\langle M \rangle} f(\mathbf{v}_l) v_r^2 cos\theta d_{max} d\phi dv_x dv_r d\theta dx, \tag{19}$$

Table 5:

	Galaxy	Cluster	Number of events N_5
M15	1.15×10^{-3}	3.4×10^{-3}	0.18
47 Tuc	5.44×10^{-4}	7.6×10^{-4}	0.14
Ter 5	1.05×10^{-2}	4.8×10^{-3}	2.45

where $\langle M \rangle$ is the mean mass of the deflecting body. We also used the passage to cylindrical coordinates, $d^3v = v_r dv_x dv_r d\theta$, and the expression for d_{max} derived from Eq. (5),

$$d_{max} = v_r(t_0 - T_0)(e^{\frac{c^3 \Delta t_{min}}{2G\langle M \rangle}} - 1)^{-1/2}. \tag{20}$$

As the velocity profile, we took a Maxwellian distribution with a velocity dispersion σ_l,

$$f(\mathbf{v}_l)d^3v = \frac{1}{\pi^{3/2}\sigma_l^3}e^{-\frac{v_l^2}{\sigma_l^2}}d^3v. \tag{21}$$

Integration over the variables $d\phi, dv_x, dv_r$ and $d\theta$ yields

$$\Gamma = 4\sigma_l^2(t_0 - T_0)\langle M \rangle^{-1}(e^{\frac{c^3 \Delta t_{min}}{2G\langle M \rangle}} - 1)^{-1/2}\int_0^{D_s}\rho(x)\,dx \tag{22}$$

As we see from Eq. (22), the event rate depends on the observing time, the velocity dispersion of the objects, their mean mass, and the matter distribution along the line of sight. Since these parameters differ significantly, we calculated the event rate Γ separately for each Galactic component and the globular cluster. The mean masses of the objects were calculated from the above mass functions; their velocity dispersions σ_l were assumed to be 210, 100, 50, and 11 km s^{-1} for the halo, bulge, disk, and the cluster, respectively (see Zasov et al. 2004; Alcobe and Cubarsi 2005; Vieira et al. 2006). The event rates are plotted against the Galactic coordinates of the source in Figs. 8 and 9 for two different heliocentric distances of the pulsar, 10.2 and 4.1 kpc (we assumed that $\Delta t_{min} = 0.5\mu$s and that the observing time $t_0 - T_0 = 5$ years). The positions of the globular clusters M15, 47 Tuc, and Terzan 5 are indicated by the crosses and the asterisk.

Table 5 gives the rates of relativistic PAT time delay events (events per year)for pulsars in the clusters listed above in the case of lensing by Galactic objects (the total value for all components)and by objects of the cluster itself and the total number of expected events N_5 for an observing time of 5 years.

Thus, by observing 22 pulsars in the globular cluster 47 Tuc, 8 pulsars in M15, and 32 pulsars in Terzan 5 for five years, one might expect ~ 3 relativistic PAT time delay events for pulsars to be detected. The pulsars in the globular cluster Terzan 5 make a major contribution to this number.

Note that the magnitude of the effect under consideration depends strongly on Δt_{min}, which is determined by the accuracy of the present-day PAT measurements and also depends

on the intensity of the specific pulsar. In particular, the expected number of relativistic PAT time delay events for pulsars in the globular cluster Terzan 5 for an observing time of 5 years decreases to $N_5 \sim 0.75$ for $\Delta t_{min} = 2.5\mu s$ and to $N_5 \sim 0.4$ for $\Delta t_{min} = 2.5\mu s$.

CONCLUSION

We considered the possibility of observing single relativistic pulsar PAT time delay events caused by the passages of massive objects close to the line of sight. Below, we briefly summarize and discuss our most interesting and important results.

(1) We determined the probabilities of single relativistic PAT time delay events for pulsars in three globular clusters, 47 Tuc, M15, and Terzan 5, caused by the passages of massive bodies of the cluster itself and Galactic objects on the path to the cluster near the line of sight. Assuming the globular cluster mass density distribution to be described by the model of an isothermal sphere with a core, we found our ratios of the probability of delay events on cluster objects to the dimensionless parameter f2 to be comparable for the high-latitude globular clusters 47 Tuc and M15 and to be slightly higher for the more compact and dense cluster Terzan 5.

(2) For the case where the massive body lies outside the cluster, we calculated the probabilities of events for two models of the mass distribution in the Galaxy: the "classical" BahcallSoneira model (Bahcall and Soneira 1980; Bahcall 1986) and the more recent model by Dehnen and Binney (1998). Our results are in good agreement with one another; a certain excess of the probabilities for the BS model is attributable to a more massive disk and a more compact bulge in this model. The ratios of the probability of the events caused by the passages of massive Galactic objects close to the line of sight to the parameter f^2 for pulsars in the globular clusters 47 Tuc and M15 are comparable to those for close passages of massive objects in the clusters themselves. At the present accuracy of measuring the expected effect from the pulsar PATs and an observing time of about 5 years, the detection probability of such events turns out to be low. The probability increases significantly only for pulsars in the globular star cluster Terzan 5, since in this case the line of sight passes through the dense bulge and disk regions.

(3) We determined the number of objects near the line of sight toward the pulsar that can produce the modulation of its PATs characteristic of the effect under consideration located both in the clusters under study and in the Galaxy on the path to the clusters. The population of brown dwarfs in the Galactic disk, whose concentration is comparable to that of the disk stars, has been taken into account for the first time. As would be expected, the number of massive objects near the line of sight is at a maximum for the globular cluster Terzan 5.

(4) We calculated the rate of relativistic pulsar PAT time delay events for the deflecting bodies in a globular cluster and the Galaxy.

The accuracy in determining the main pulsar parameters, such as the first, $\dot{p}$, and second,

$\ddot{p}$, period derivatives, by processing the timing data affects significantly the possibility of detecting small relativistic effects. For most of the pulsars in globular star clusters, only the first period derivative has been measured well, $\sim 10^{-20}$ (the second period derivative has been measured only for some of the sources, $\sim 10^{-31}$ s^{-1}; see, e.g., the catalog of pulsars and their parameters at http://www.atnf.csiro.au). As regards the accuracy in determining $\dot{p}$, it is presently $\sim 0.1\%$ (see, e.g., Freire et al. 2003) for the globular cluster 47 Tuc. The changes in $\dot{p}$ due to the effect under consideration are (several) $\times 10^{-25} - 10^{-24}$ for the massive bodies passing near the "event tube" boundary. In reality, this may prove to be considerably larger because of the bodies passing closer to the line of sight (depends as $\propto d^{-2}$ and may be comparable to or even higher than the current accuracy in determining $\dot{p}$; see also Wex et al. 1996).

All of the results obtained in this paper lead us to conclude that the effect under consideration is small and difficult to measure at the current level of observations for faint sources, which the pulsars in globular clusters are; allowing for this effect will become increasingly important with improving measurement accuracy. However, this effect may turn out to be measurable for bright pulsars even now at a pulsar PAT measurement accuracy of ~ 100 ns in observing intervals of several years.

As was noted in the Introduction, invisible dark matter particles are capable of clumping into compact objects with a mass of the order of the Earth mass (Berezinskiy et al. 2003) and their detection is of great interest to researchers. However, according to our calculations, at the current level of accuracy, such objects cannot be detected based on pulsar PAT measurements in a "reasonable" observing time (~ 10 years) because of their low mass (see Eq. (5)).

In conclusion, note that, apart from the Shapiro effect considered here, the behavior of the pulsar PAT residuals can also be affected by other effects, such as close passages of stars in globular clusters and variations in the interstellar medium. The latter effect depends on the frequency at which the observations are performed and can be taken into account by performing observations at several frequencies. Close passages of stars in globular clusters can cause significant changes in both pulsar trajectories and periods, $\Delta p/p \sim 10^{-8}$. However, the probability of such events is low, because the mean distance between the cluster stars is large, ~ 0.1 pc (see, e.g., Rodin 2000).

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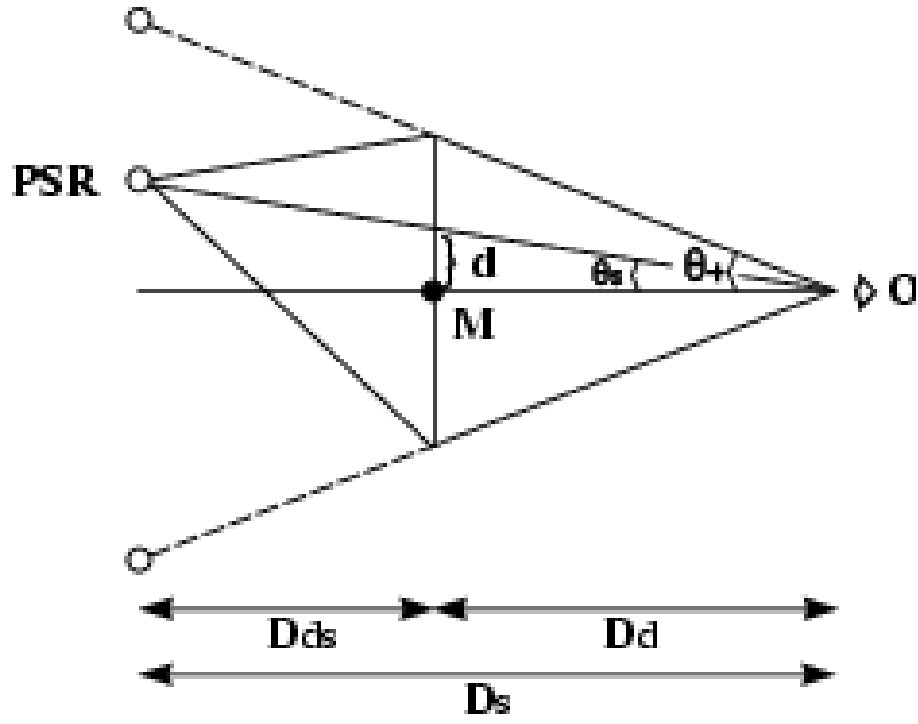46. A. V. Zasov, A. V. Khoperskov, and N. V. Tyurina, Pisma Astron. Zh. 30, 653 (2004)[Astron. Lett. 30, 593 (2004)].

Fig. 1: Geometry of the problem under consideration: O - observer, PSR - pulsar, - gravitating mass. For the remaining notation, see the text.
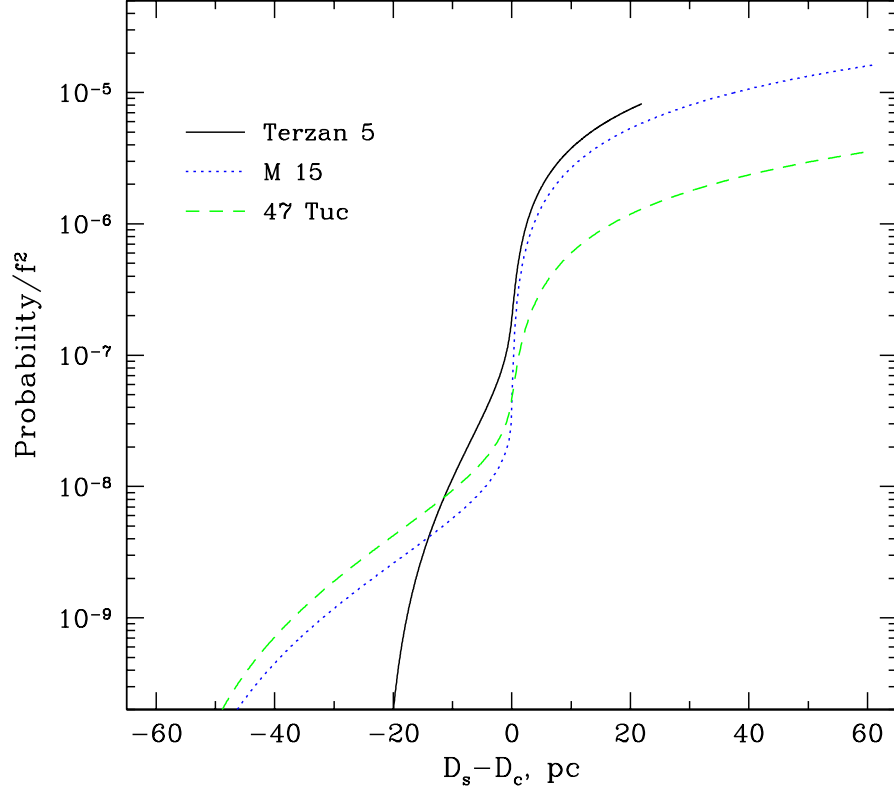
Fig. 2: Ratio of the detection probability of a time delay event to f^2 as a function of pulsar position in the globular clusters M15 (dotted line), 47 Tuc (dashed line), and Terzan 5 (solid line). All of the sources are located along the observers line of sight to the cluster center, but at different radial distances from the cluster center.
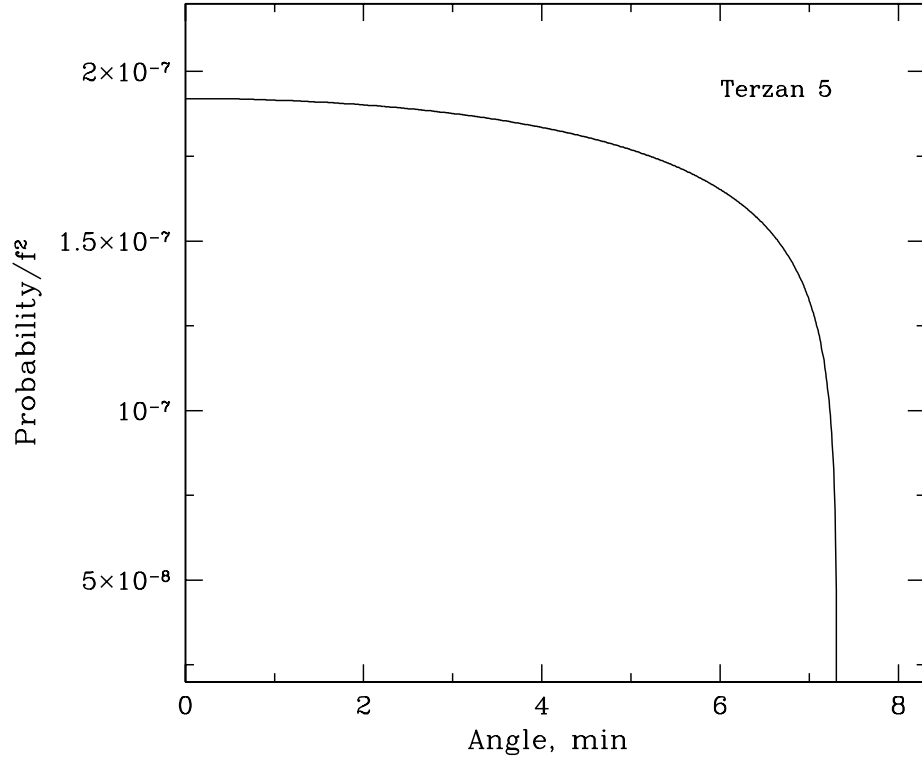
Fig. 3: Ratio of the detection probability of a time delay event to f^2 as a function of pulsar position in the cluster Terzan 5. All of the sources are located at the distance of the cluster center, but at different angular distances from its center.
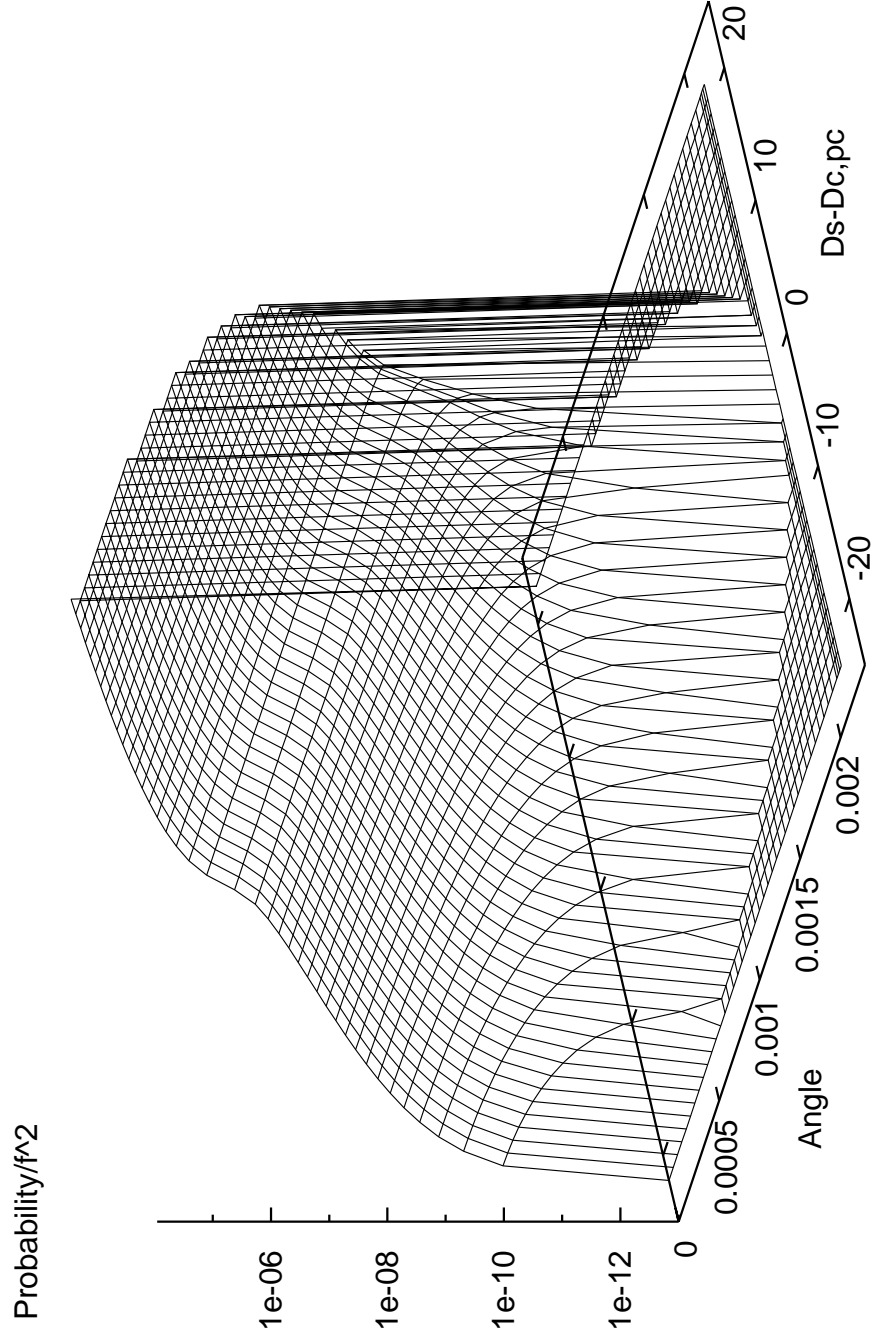
Fig. 4: 3D distribution of the ratio of the detection probability of a time delay event to f^2 for a source in the globular cluster Terzan 5 as a function of its radial (pc) and angular (rad) distances to the cluster center.
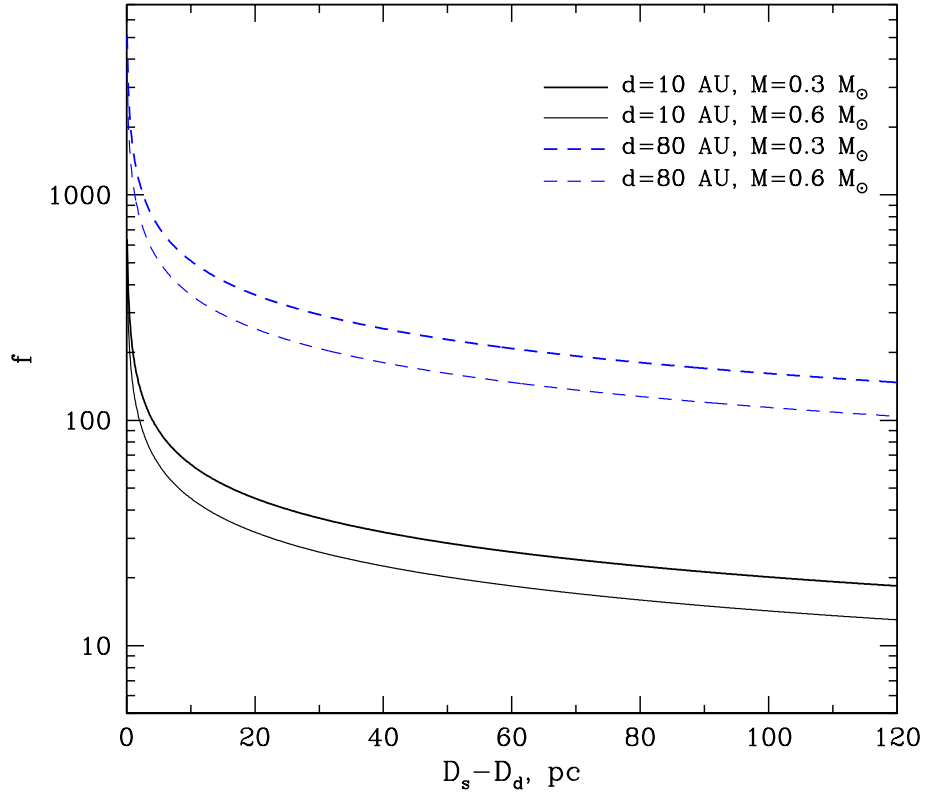
Fig. 5: Dimensionless impact parameter f as a function of the distance between the deflecting body with a mass of $0.3M_\odot$ and $0.6M_\odot$ and the pulsar for two different impact parameters, $d = 10$ and $d = 80$ AU.
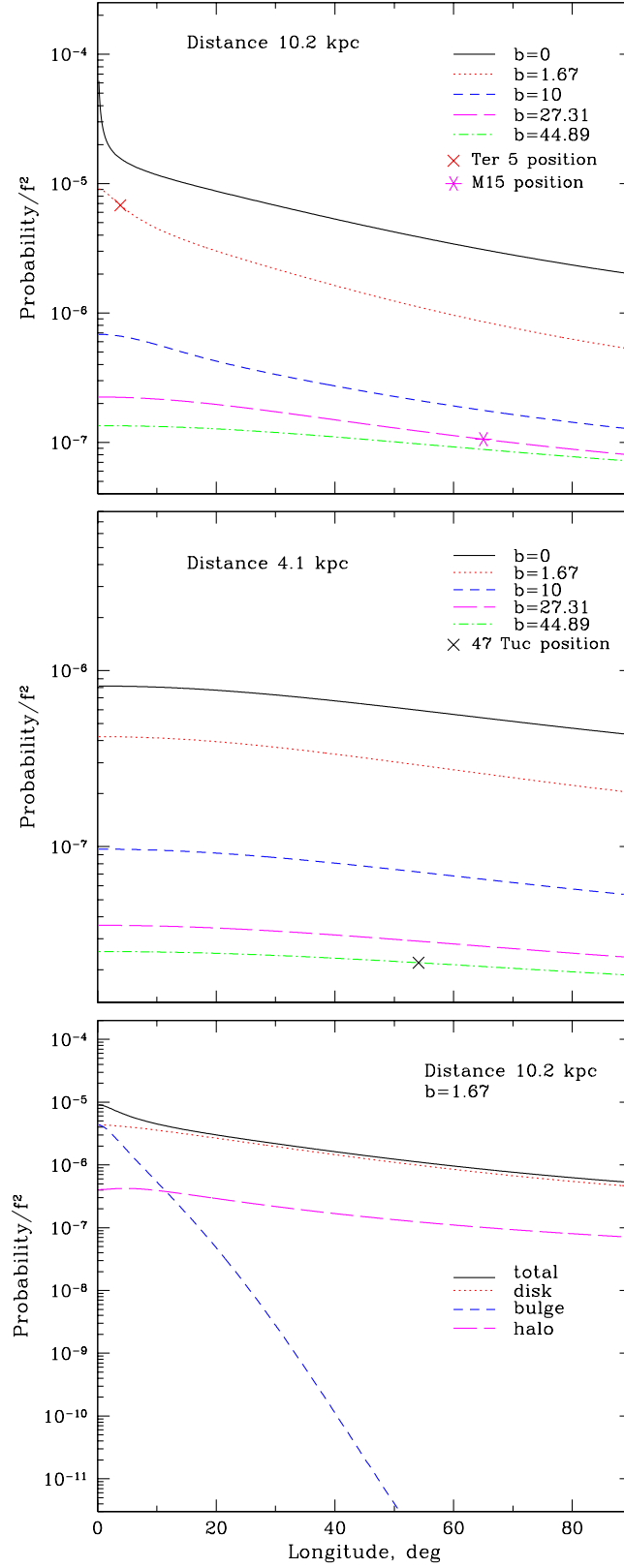
Fig. 6: Ratio of the detection probability of a time delay event to f^2 as a function of Galactic longitude l and latitude b of a source at a distance of (a) 10.2 and (b) 4.1 kpc. (c) The relative contributions from various Galactic components to the total lensing probability for a source at a latitude of 1.67°. The positions of the globular clusters Terzan 5, M15, and 47 Tuc are indicated by the crosses and the asterisk. The DB model of the Galaxy is used.
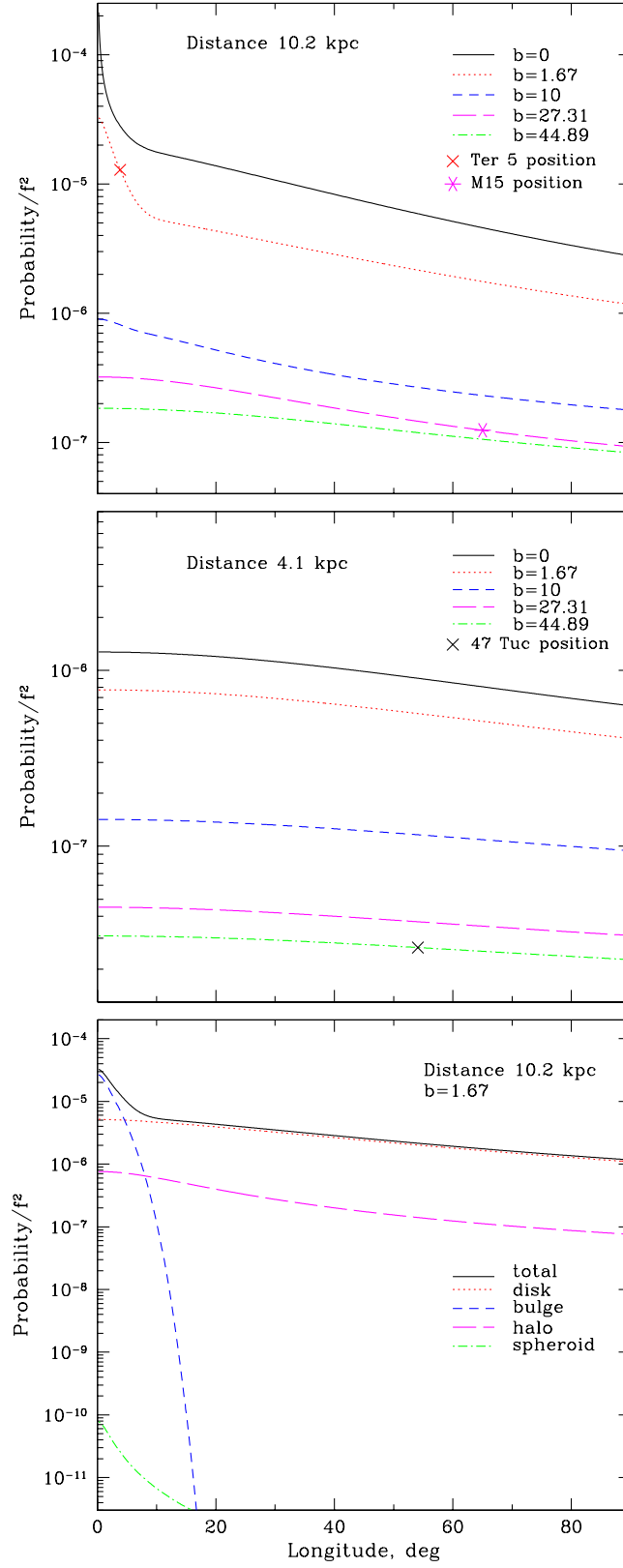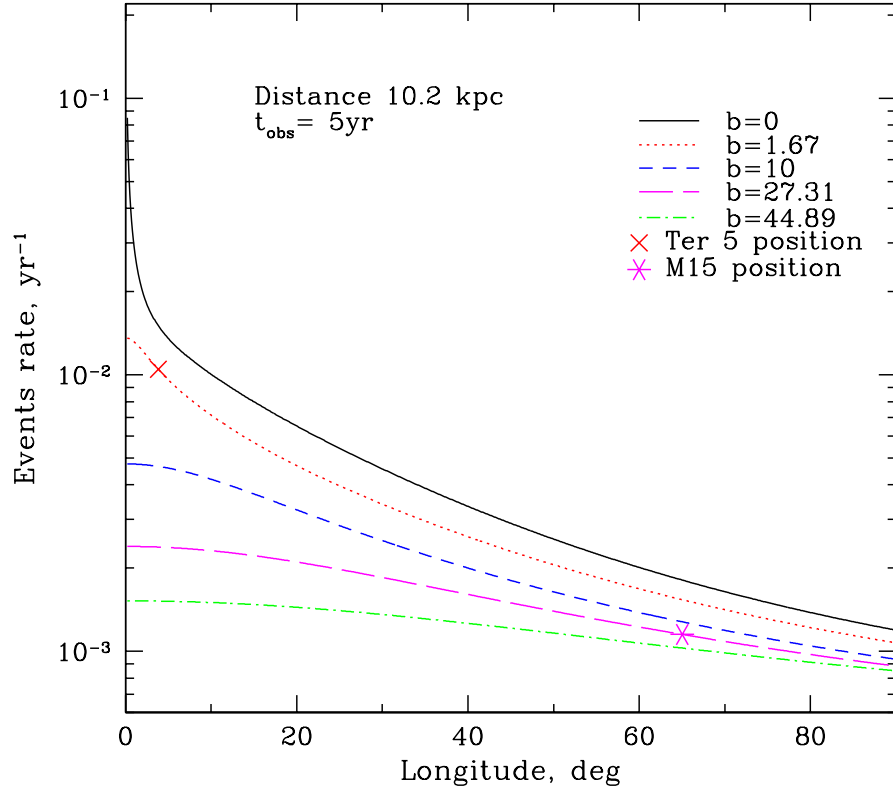
Fig. 7: Same as Fig. 6 for the BS model of the Galaxy.

Fig. 8: Event rate as a function of Galactic longitude l and latitude b of a pulsar at a distance of 10.2 kpc. The positions of the globular clusters Terzan 5 and M15 are indicated by the cross and the asterisk, respectively. The DB model of the Galaxy is used.
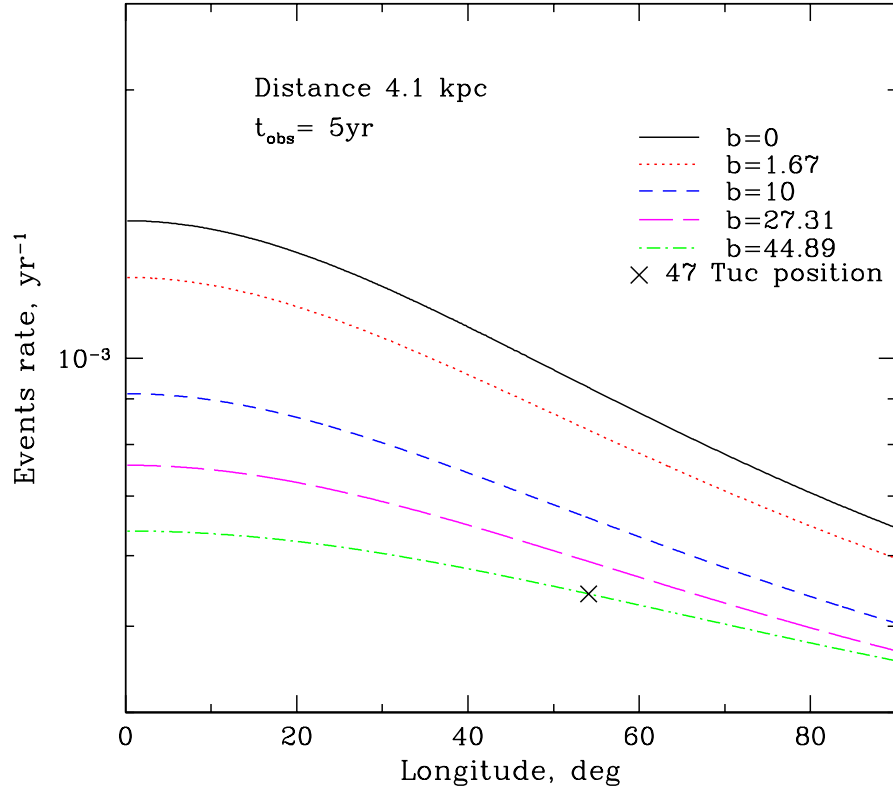
Fig. 9: Event rate as a function of Galactic longitude l and latitude b of a pulsar at a distance of 4.1 kpc. The position of the globular cluster 47 Tuc is indicated by the cross. The DB model of the Galaxy is used.